Q1. Prove that the sum of the potential energy and the kinetic energy of a particle with mass m under the force $\boldsymbol{F}$ is a constant. In mechanics the potential energy V is defined as $\boldsymbol{F} = -\nabla V$ and the kinetic energy K is defined as $K = \frac{1}{2}mv^2$ where v is velocity.

Q2. Consider the Maxwell's Equations in space where there is no charge or current:

$$\nabla \cdot \boldsymbol{E} = 0 \qquad \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \nabla \cdot \boldsymbol{B} = 0 \qquad \nabla \times \boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}$$

Where $\boldsymbol{E}$ is the electric field, $\boldsymbol{B}$ is the magnetic field, c is the speed of light and t is time. Show that both fields $\boldsymbol{E}$ and $\boldsymbol{B}$ satisfy the Wave Equation $\frac{\partial^2 \boldsymbol{F}}{\partial t^2} = k^2 \nabla^2 \boldsymbol{F}$ where $\boldsymbol{F}$ is the field and k is a constant.

Hint: Use $\nabla \times (\nabla \times \boldsymbol{F}) = \nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2 \boldsymbol{F}$ without proof. Here $\nabla^2 \boldsymbol{F} = (\nabla \cdot \nabla)\boldsymbol{F}$

Q3. Magnetic field $\boldsymbol{B}$ induced at the origin by a curve C (given by the position vector) carrying a steady current I in the direction of the curve C is given by the Biot-Savart Law:

$$\boldsymbol{B} = \frac{\mu_0 I}{4\pi} \int_C \frac{\boldsymbol{r} \times \boldsymbol{dr}}{\|\boldsymbol{r}\|^3}$$

Here μ_0 is the permeability of the vacuum. Find the magnetic field when the curve C is the circle with radius a and center at the origin with the direction of the curve is anticlockwise.

Q4. Use Divergence theorem $\iint_S \boldsymbol{F}\cdot d\boldsymbol{S} = \iiint_V \mathrm{div}\boldsymbol{F}\, dV$ to prove the Archimedes's Principle(upward buoyant force that is exerted on a body immersed in a fluid is equal to the weight of the fluid that the body displaces). **Hint**: Fluid force is normal to the surface.