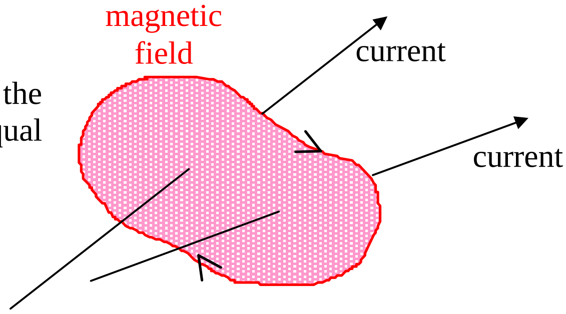
Electromagnetic Theory - *J. R. Lucas*

One of the basic theorems in electromagnetism is the Ampere's Law which relates, the magnetic field produced by an electric current, to the current passing through a conductor.

Ampere's Law



Ampere's Law states that the line integral of the magnetic field ***H*** taken around a closed path is equal to the total current enclosed by the path.

i.e. $\oint H \cdot dl = \sum I$

For a uniform field, ***H*** is a constant and we have

$$H \, . \, l = \Sigma \; I$$

or if ***H*** is constant over sections, with different sections having different ***H***,

then $\Sigma H \, . \, l = \Sigma I$

Magnetic Field

The magnetic field at a point is defined as being equal to the force acting on a unit magnetic pole placed at that point.

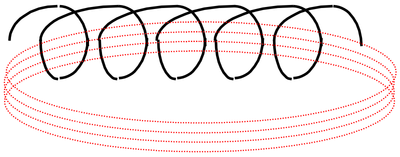
[Unit of magnetic field is ***ampere per meter (A/m)***]

Magnetomotive force (mmf)

Magnetomotive force is the flux producing ability of an electric current in a magnetic circuit. [It is something similar to electromotive force in an electric circuit].

[Unit of magnetomotive force is ***ampere (A)***] - *Note:* Although some books use the term *ampere-turns*, it is strictly not correct as *turns* is not a dimension]

mmf $\Im = \Sigma I$



Consider a coil having ***N*** turns as shown.

It will link the flux path with each turn, so that

total current linking with the flux would be

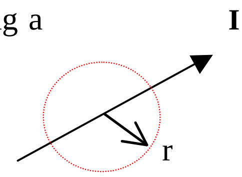
$$\Sigma I = N.I$$

Thus from Ampere's Law, the **mmf** produced by a coil of ***N*** turns would be ***N I***,

and $N \, I = H \, l.$

Field produced by a long straight conductor

If a circular path of radius ***r*** is considered around the conductor carrying a current ***I***, then the field $\boldsymbol{H_r}$ along this path would be constant by symmetry.

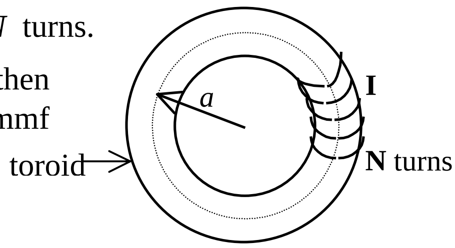


∴ by Ampere's Law, $1.I = H_r . 2\pi \, r$

or $\boldsymbol{H_r} = \dfrac{\boldsymbol{I}}{2\pi \, \boldsymbol{r}}$ at a radial distance ***r*** from the conductor.

Field produced inside a toroid

Consider a toroid (similar to a ring) wound uniformly with ***N*** turns.



If the mean radius of the magnetic path of the toroid is ***a***, then the magnetic path length would be $2\pi.a$, and the total mmf produced would be ***N I***.

Thus from Ampere's Law

magnetic field $\boldsymbol{H} = \dfrac{\boldsymbol{NI}}{2\pi\, \boldsymbol{a}}$ inside the toroid. [variation of the magnetic field inside the cross section of the toroid is usually not necessary to be considered and is assumed uniform].

Magnetic flux density

The magnetic field ***H*** gives rise to a magnetic flux ϕ, which has a magnetic flux density ***B*** for a given area ***A***. The relationship between ***B*** and ***H*** is given by the permeability of the medium μ..

$\boldsymbol{B} = \mu \boldsymbol{H}$, where $\mu = \mu_o \mu_r$,

μ_r is the relative permeability and μ_o is the permeability of free space

$\mu_o = 4\pi \times 10^{-7}$ H/m

[permeability of air is generally taken to be equal to that of free space in practice]

[Unit of permeability is ***henry per meter (H/m)***].

[Unit of magnetic flux density is the ***tesla (T)***]

$$\phi = B.A$$

[Unit of magnetic flux is the ***weber (Wb)***]

Reluctance of a magnetic path

A magnetic material presents a Reluctance ***S*** to the flow of magnetic flux when an mmf is applied to the magnetic circuit.

[This is similar to the resistance shown by an electric circuit when an emf is applied]

Thus ***mmf = Reluctance*** $\times$ ***flux*** or $\Im = S\,.\,\phi$

For a uniform field, $\Im = N\,I = H.l$, and $\phi = B.\,A = \mu\, H.\,A$

$\therefore\ H.l = S\,.\,\mu\, H.\,A$

so that the magnetic reluctance $\boldsymbol{S} = \dfrac{\boldsymbol{l}}{\mu\, \boldsymbol{A}}$, where ***l*** = length and ***A*** = cross-section

[Unit of magnetic reluctance is ***henry***$^{-1}$ ($\boldsymbol{H}^{-1}$)]

Magnetic Permeance ***Λ*** is the inverse of the magnetic reluctance. Thus $\Lambda = \dfrac{\mathbf{1}}{\boldsymbol{S}} = \dfrac{\mu\, \boldsymbol{A}}{\boldsymbol{l}}$

[Unit of magnetic permeance is ***henry (H)***]

Self Inductance

While the reluctance is a property of the magnetic circuit, the corresponding quantity in the electrical circuit is the inductance.

Induced emf $\boldsymbol{e} = \boldsymbol{N}\dfrac{d\phi}{dt} = \boldsymbol{L}\dfrac{d\,\boldsymbol{i}}{dt}$, $\quad \boldsymbol{N}\,\phi = \boldsymbol{L}\,\boldsymbol{i}$, $\quad \boldsymbol{L} = \dfrac{N\phi}{i}$

The self inductance $\boldsymbol{L}$ of a winding is the flux linkage produced in the same winding due to unit current flowing through it.

For a coil of $\boldsymbol{N}$ turns, if the flux in the magnetic circuit is ϕ, the flux linkage with the coil would be $\boldsymbol{N}.\phi$.

also since $\boldsymbol{N}\,\boldsymbol{I} = \boldsymbol{S}\,\phi$, $\boldsymbol{L} = \dfrac{N^2}{S} = \dfrac{N^2\mu\,A}{l}$

Thus the inductance of a coil of $\boldsymbol{N}$ turns can be determined from the dimensions of the magnetic circuit.

Mutual Inductance

When two coils are present in the vicinity of each other's magnetic circuit, mutual coupling can take place. One coil produces a flux which links with the second coil, and when a current in the first coil varies, an induced emf occurs in the second coil.

Induced emf in coil 2 due to current in coil 1:

$$e_2 = N_2\frac{d\phi_{12}}{dt} = M_{12}\frac{d\,i_1}{dt},\quad N_2\phi_{12} = M_{12}i_1,\quad M_{12} = \frac{N_2\phi_{12}}{i_1}$$

The Mutual inductance $\boldsymbol{M}_{12}$, of coil 2 due to a current in coil 1, is the flux linkage in the coil 2 due to unit current flowing in coil 1.

also since $\boldsymbol{N_1}\,\boldsymbol{I_1} = \boldsymbol{S}\,\phi_1$, and a fraction $\boldsymbol{k_{12}}$ of the primary flux would link with the secondary, $\phi_{12} = \boldsymbol{k_{12}}\,.\,\phi_1$

$$\therefore M_{12} = \frac{k_{12}N_1N_2}{S} = \frac{k_{12}N_1N_2\mu\,A}{l},$$

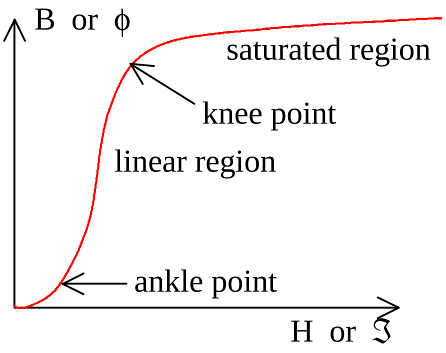
$\boldsymbol{k_{12}}$ is known as the coefficient of coupling between the coils.

$\boldsymbol{k_{12}} = \boldsymbol{k_{21}}$ so that $\boldsymbol{M_{12}} = \boldsymbol{M_{21}}$. For good coupling, $\boldsymbol{k_{12}}$ is very nearly equal to unity.

Magnetisation Characteristic

In the analysis so far, the relationship between B and H has been assumed to be linear. That is the permeability has been assumed to be constant. This is not the case with magnetic materials, due especially to saturation.

The curve drawn shows a typical shape of the magnetisation characteristic of a magnetic material. They are characterised by an ankle point for very low levels of flux density, a linear region and a saturation region. The knee point appears between the linear region and the saturated region. In the linear region, the permeability is much greater than that of free space, whereas in the saturated region it is close to that of free space. The characteristic may be drawn with either B or ϕ plotted on the y-axis and H or $\Im$ on the x-axis.



The terms, *ankle* point and *knee* point are used to define the curve as the diagram looks quite a bit like a bent leg with the ankle and the knee occurring in those positions.

Magnetic materials are usually operated in their linear region, and for best utilisation of the material, they are usually operated just below the knee point. [Maximum operating flux densities in steel are in the region of 1.6 T].

Since permeability is the ratio of B to H, it is no longer constant for a non-linear magnetisation characteristic. In these regions an incremental permeability is defined which corresponds to the rate of change of B with H.

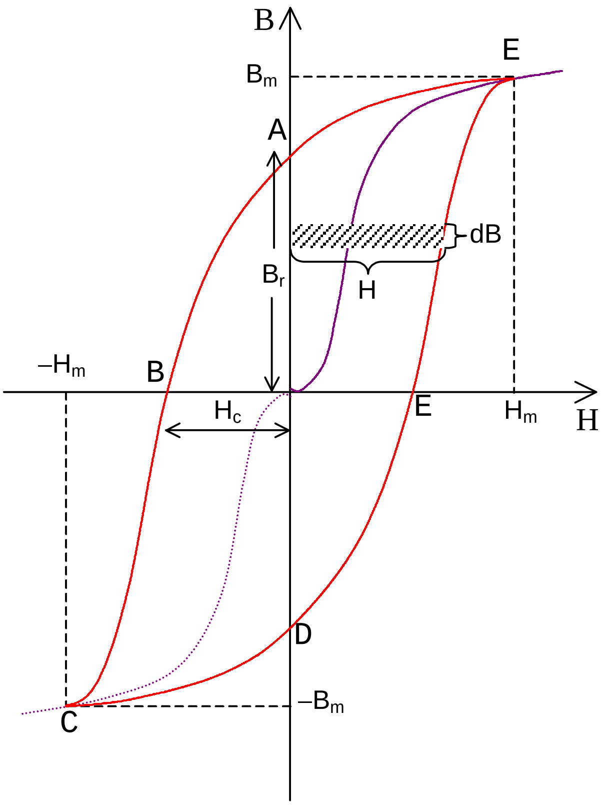
$$\text{Incremental permeability} \quad = \frac{dB}{dH}$$

$$\text{Absolute permeability} \quad \mu_r \quad = \frac{B}{H}$$

For magnetic materials, the relative permeability in the linear region can vary from about 2,000 to about 100,000 dependant on the material.

B-H loop

If the magnetic field is increased, at a certain stage the magnetic flux shows negligible increase. This situation is saturation.



If the magnetic field is applied to an initially demagnetised magnetic material and then subsequently removed, the material retains some of the magnetisation. That is, the magnetic flux density produced by the magnetic field does not completely vanish. The amount of remaining flux density is known as the *Remnance* B_m. This remnance can be removed by an application of a magnetic field in the opposite direction. The amount of demagnetising magnetic field required is known as the *coercivity* (or coercive force) H_c.

This of course must mean that the demagnetisation is not occurring along the original magnetisation curve. An increase in the magnetic field in the negative direction would result in saturation in the reverse direction.

If the process is continued, a loop would be formed. This loop is known as the B-H loop or the hysteresis loop. This loop normally occurs when the magnetic field is alternating and is associated with a loss in energy, known as the hysteresis loss.

This may be calculated as follows.

power supplied to the magnetic field $\quad P = e.i$

but from the laws of electromagnetic induction, $\quad e = N\dfrac{d\phi}{dt}$

so that $P = e.i = N\dfrac{d\phi}{dt}.i = \dfrac{dB}{dt}.A.N.i = \dfrac{dB}{dt}\cdot A\cdot H\cdot l$

i.e. $P = \dfrac{dB}{dt}\cdot H\cdot volume$

$\therefore$ *energy supplied per unit volume* $= \int H\cdot\dfrac{dB}{dt}\cdot dt = \int H\cdot dB$

[Note: $\int$ H.dB is also the area of the elemental area indicated on the B-H loop]

Thus *energy supplied/unit volume/cycle* $= \oint H\cdot dB =$ *area of B-H loop*

This energy loss is known as the hysteresis loss.

Thus the energy supplied/unit volume/second or *power* would be the above multiplied by cycles/second or *frequency*.

Thus power loss due to hysteresis P_h = area of B-H loop $\times$ frequency

i.e. $P_h \propto f$

It is also seen that the area of the loop increases as the maximum flux density B_m reached increases. The increase in B_m also causes a corresponding increase in H_m, but not proportional to the increase in B_m.

Thus the hystesis loss would also be proportional to a power of B_m.

Steinmetz Law

Steinmetz Law states that the energy loss per cycle due to hysteresis is proportional to B_m^n.

Generally, a $\cong$1.6.

i.e. energy loss/unit volume/cycle = area of hysteresis loop = $\eta.\ B_m^{a}$

where the constant η is known as the Steinmetz constant and the index *a* is known as the Steinmetz index.

Thus for an alternating supply with frequency f,

power loss/unit volume = $\eta.\ B_m^{1.6}.f$

The Steinmetz constant for some common magnetic materials are

Hard cast steel 7000, cast steel 750 to 3000, cast iron 2760 to 4000,very soft iron 500, silicon sheet steel (with 0.2% Si) 530, silicon sheet steel (with 4.8% Si) 191.

Energy stored in a magnetic field

Energy stored in an inductor = $\int \boldsymbol{v\cdot i\cdot dt} = \int \boldsymbol{L\dfrac{di}{dt}\cdot i\cdot dt} = \int \boldsymbol{L\cdot i\cdot di} = \tfrac{1}{2}\boldsymbol{Li^2}$

Energy stored in a unit volume in magnetic field

$$= \int \frac{\boldsymbol{N\cdot \frac{d\phi}{dt}\cdot i\cdot dt}}{\text{volume}} = \int \frac{\boldsymbol{N\cdot i\cdot d\phi}}{\boldsymbol{A.l}} = \int \frac{\boldsymbol{N\cdot i}}{\boldsymbol{l}}\cdot \boldsymbol{d}\frac{\boldsymbol{\phi}}{\boldsymbol{A}} = \int \boldsymbol{H\cdot dB}$$

$$= \int \frac{\boldsymbol{B}}{\mu}\cdot \boldsymbol{dB} = \frac{1}{2}\frac{\boldsymbol{B}^2}{\mu} = \frac{1}{2}\boldsymbol{B\cdot H} \qquad \text{J/m}^3$$

Force exerted in an magnetic field

Consider moving the electromagnet so that the spacing changes by ***dx***

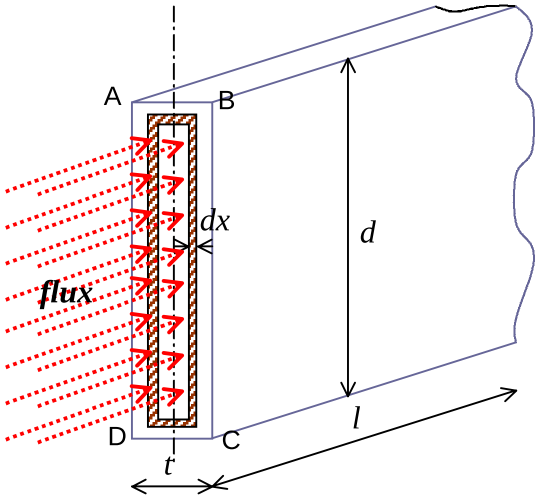
change in energy stored = ½ ***B.H*** × (change in volume) = ½ ***B.H*** × ***A. dx***

Also, change in energy stored = work done = ***F . dx***,

∴ ***F . dx = ½ B.H . A. dx*** or ***F = ½ B.H . A***

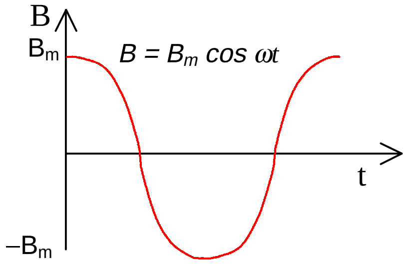
i.e. Force exerted on unit area in an electric field = ***F/A*** = ½ ***B.H*** N/m^2

Eddy Current Loss



The term eddy current is applied to an electric current which circulates within a mass of conductor material, when the material is situated in a varying magnetic field. These eddy currents result in a loss of power. Eddy current losses together with the hysteresis losses cause heating of magnetic materials. To reduce these losses, not only are materials with low Steinmetz coefficients chosen but the magnetic core is made in the form of laminations to reduce the eddy current loss.

Consider the magnetic flux incident on the face ABCD of the cross-section of a lamination of thickness *t*. [The thickness of the lamination has been grossly exagerated to show details of eddy current paths].



If the flux is alternating with flux density B = B_m cos ωt, then eddy currents will establish themselves round the periphery, as shown shaded on the diagram.

The eddy currents produced will also strive to change the flux, so that the flux at the periphery would be comparatively more. However, as very thin plates are considered (fraction of a mm), this effect can be neglected.

Consider an eddy current path at a distance *x* from the centre-line of the cross-section and penetrating the full length *l* of the plates.

If the frequency of the supply is *f* and a elemental path of thickness *dx* is considered, then the average rate of change of flux density over half a cycle would be given by difference in the positive and negative peaks divided by the time for half a cycle.

i.e. average $\frac{dB}{dt}$ over half cycle $= \frac{B_m - (-B_m)}{\frac{1}{2}T} = \frac{4B_m}{T} = 4\, B_m f$

∴ average rate of change of flux $= 4\, B_m f \,.\, area$

For a given path, at distance *x*, the area would be the area enclosed by that path.

∴ average rate of change of flux $= 4\, B_m f \,.\, 2x \,.\, d = 8\, B_m \,.\, f \,.\, x \,.\, d$

i.e. average induced emf in the eddy current path $= 8\, B_m \,.\, f \,.\, x \,.\, d$

and, rms induced emf in the eddy current path $= 8\, B_m \,.\, f \,.\, x \,.\, d \,.\, k = e$

where *k* is the form factor [for a sinusoidal waveform, k = 1.111]

Since the thickness of the lamination is negligible, the length of each eddy current path can be considered as constant and equal to *2d*, and the area through which current flows as *dx.l*

$\therefore$ resistance of eddy current path = $\dfrac{\rho.2d}{l.dx}$ = r,   where ρ is the resistivity of the material

$\therefore$ power loss in elemental strip due to eddy currents = $\dfrac{e^2}{r} = \dfrac{(8B_m f\,x d\,k)^2}{\dfrac{\rho.2d}{l.dx}}$

$$= \frac{32B_m^2 f^2 x^2 d\,k^2 l.dx}{\rho}$$

$\therefore$ total power loss in plate of thickness t = $\displaystyle\int_0^{\frac{t}{2}} \frac{32B_m^2 f^2 x^2 d\,k^2 l}{\rho}.dx = \frac{4}{3}\frac{B_m^2 f^2 t^3 d\,l k^2}{\rho}$

volume of the plate = *d.l.t*, so that

$$\text{Eddy current loss per unit volume} = \frac{4}{3}\frac{B_m^2 f^2 t^2 k^2}{\rho}$$

It is thus seen that the eddy current loss per unit volume is proportional to the square of the thickness of the individual plates. Thus for a given volume, if the thickness of the laminations is made very small, the eddy current losses can be minimised.

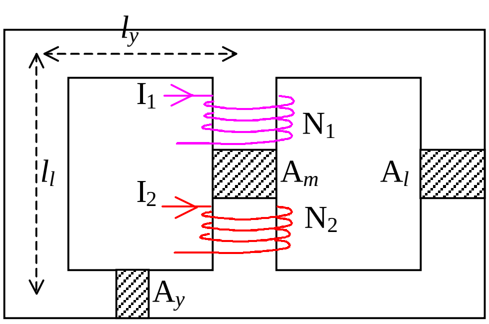
Comparison of properties of eddy current and hysteresis losses

1. Eddy current loss is proportional to the square of the frequency, while the hysteresis loss is proportional directly to the frequency.
2. Eddy current loss is proportional to the square of the peak flux density, while the hysteresis loss is usually proportional to the 1.6^{th} power of the peak flux density.
3. Eddy current loss is proportional to the square of the thickness of the laminations while the hysteresis loss does not depend on thickness of laminations.
4. Eddy current loss is dependant on the resistivity of the material, while the hysteresis loss is dependant on the Steinmetz constant of the material.

Analysis of Electromagnetic Circuits

Electromagnetic circuits can be analysed in a manner similar to the analysis of resistive circuits.

Consider the following two winding transformer wound on a three limb core.



Cross section areas of the core, and the effective lengths of magnetic paths are indicated.

It is assumed that the cross-section does not change at the corners.

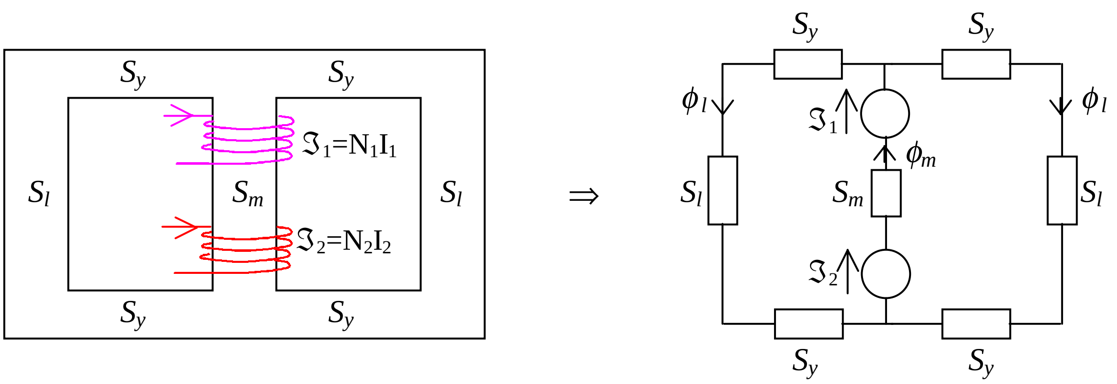
m.m.f.s $\Im_1$ and $\Im_2$ are produced in the two windings and equal to N_1I_1 and N_2I_2 .

reluctances of each outer limb $S_l = \dfrac{l_l}{\mu_o \mu_r A_l}$,

reluctance of each part of top and bottom yokes $S_y = \dfrac{l_y}{\mu_o \mu_r A_y}$,

reluctances of middle limb $S_m = \dfrac{l_l}{\mu_o \mu_r A_m}$, [length of middle limb same as outer]

If the fluxes flowing in the paths are ϕ_l (outer limbs, yokes) and ϕ_m (centre limb), then an equivalent circuit similar to the electrical equivalent circuit may be drawn as follows.



The fluxes can be calculated using laws similar to Ohm's law and Kirchoff's law as follows.

$\phi_m = \phi_l + \phi_l$ similar to Kirchoff's current law

$\Im_1 + \Im_2 = S_m\,\phi_m + (2S_y + S_l)\,\phi_l$ similar to Kirchoff's voltage law and Ohm's law

Only one loop was considered as both outer limbs are identical and must therefore have the same flux. If the outer limbs were different, then there would have been one additional flux term and one additional equation.

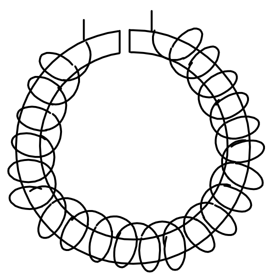
The only unknowns are ϕ_m and ϕ_l which can be calculated.

In the case of three phase transformers, the winding currents would have different phase angles, so that the corresponding mmfs too would have different phase angles. The analysis of this would be similar to the analysis of three phase problems, but no equivalent being there for inductances and capacitances in the corresponding equations.

The above analyses are valid only in the linear region of the magnetisation characteristic where the permeability can be assumed to be constant. However, when saturation occurs, the analysis is more complicated.

Analysis in the presence of a non-linear magnetisation characteristic

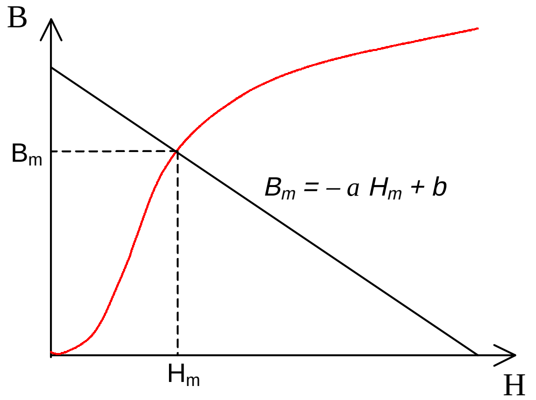
Only a simple circuit having a non-linear magnetic characteristic and a series air gap will be considered to illustrate the method of analysis. It is assumed that there is no fringing of flux around the air gap so that the flux density will be the same in both the air gap as well as the magnetic core. $B_m = B_a$



The characteristic of the magnetic core is also known.

The air gap has a linear characteristic with permeability μ_o.

Let the cross section the core (and air gap) be A, the length of the magnetic path be l_m in the magnetic material and l_a in the air gap.



Let the number of turns in the coil be N and the current in the winding be I.

Then from Ampere's law

$$N\,I = H_m\,l_m + H_a\,l_a$$

for the air gap, $H_a = \dfrac{B_a}{\mu_o} = \dfrac{B_m}{\mu_o}$

$$\therefore \quad N\,I = H_m\,l_m + \frac{B_m}{\mu_o}\,l_a, \quad or \quad B_m = -\,a\,H_m + b$$

Since this equation has been written in terms of the parameters of the magnetic material, intersection of this straight line with the magnetisation characteristic would give the operating position.