2.0 Network Theorems

2.1 Basic Concepts

The fundamental theory on which many branches of electrical engineering, such as electric power, electric machines, control, electronics, computers, communications and instrumentation are built is the Electric circuit theory. Thus it is essential to have a proper grounding with electric circuit theory as the base. An electric circuit is the interconnection of electrical elements.

2.1.1 Terminology

The most basic quantity in an electric circuit is the *electric charge **q***. The law of *conservation of charge* states that charge can neither be created nor destroyed. Thus the algebraic sum of the charges in a system does not change. The charge on an electron is -1.602×10^{-19} C. [Unit of electric charge is the *coulomb (C)*].

The rate of flow of electric charges or electrons constitute an *electric current **i***. By convention (a standard way of describing something so that everyone understands the same thing), the electric current flows in the opposite direction to the electrons. [Unit of electric current is the *ampere (A)*].

$i = \dfrac{dq}{dt}$ and the charge transferred between time t_o and t is given by $q = \int_{t_o}^{t} i\, dt$

To move an electron in a conductor in a particular direction, or to create a current, requires some work or energy. This work is done by the *electromotive force (emf)* of the source or the *potential difference*. This is also known as *voltage difference* or *voltage* (with reference to a selected point such as *earth*).

The voltage v_{ab} between two points *a* and *b* is the *energy* (or work) w required to move a unit charge q from ***a*** to ***b*****.** [Unit of voltage is the *volt (V)*]

$v_{ab} = \dfrac{dw}{dq}$, [*Note:* The suffixes ***ab*** need not be written when there is no ambiguity (more than one meaning)

In addition to current and voltage, we need to know the *power **p*** handled by an electric circuit. Power is the rate of doing work or transferring energy. [Unit of power is the *watt (W)*]

Thus $p = \dfrac{dw}{dt} = \dfrac{dw}{dq}.\dfrac{dq}{dt} = v.i$ i.e. $p = v.i$

Energy is the capacity to do work. [Unit of energy is the *joule (J)*]

The energy transferred from time t_o and t is given by $w = \int_{t_o}^{t} p\, dt = \int_{t_o}^{t} v.i\, dt$

An Analogy (*a similar example)*

Consider a tap in the garden supplied from an overhead tank. It has a certain potential energy mgh joule. We can also say that it has a potential of h metre. [This is similar to saying we have a battery with a certain energy capacity $E\ it$ and having a potential (or emf) of E**.** for example, a car battery with an emf of **12 V** and a capacity of **60 Ah** or approximately **12×60×3600 J**]. If we consider two tanks of the same height, but different capacity, they would have the same potential but different capacity depending on the volume of the tank corresponding to ***mg*****.** [Similarly different batteries could have the same emf (or potential) but have different capacities. For example, a "12 V car battery" and "8 pen-torch batteries connected in series" would have the same emf, but obviously a completely different energy capacity]. Depending on how much we open the tap (changing the resistance to water flow of the path), the water will come out at different rate. [This is similar to connecting a battery to a circuit, and depending on the resistance of the circuit the current coming out will differ.] The maximum pressure available at the tap is when the flow is a minimum, and there is no head loss due to friction in the pipe, and this corresponds to the potential of the tank h. We can never get a pressure of more than h (except momentarily when we perhaps put our finger to partly block the flow of water). [Similarly, the maximum potential that is available to a load connected is E when no current is taken out of the battery (open circuit); and, there is no voltage drop in the internal resistance of the battery and wire resistance since there is no current. We can never get a potential of more than E (except during a transient operation, and inductance and/or capacitance is there in the circuit)]. The water coming out from the tap could either be (a) absorbed by the ground (when it is lost) or (b) collected in a bucket (where it is stored and can be put back into the tank). This is similar to the current going into a (a) resistor in which the energy gets dissipated (or lost) as heat to the surroundings and (b) either an inductor or capacitor where energy is stored in electromagnetic or electrostatic form and which can be retrieved later and is not lost.

2.1.2 Basic Circuit Elements

Electric Circuits consist of two basic types of elements. These are the *active elements* and the *passive elements*.

An ***active element*** is capable of generating electrical energy. [In electrical engineering, generating or producing electrical energy actually refers to conversion of electrical energy from a non-electrical form to electrical form. Similarly energy loss would mean that electrical energy is converted to a non-useful form of energy and not actually lost. - *Principle of Conservation of Mass and Energy*].

Examples of active elements are *voltage source* (such as a battery or generator) and *current source*. Most sources are independent of other circuit variables, but some elements are *dependant* (modelling elements such as transistors and operational amplifiers would require dependant sources).

Active elements may be ***ideal*** voltage sources or current sources. In such cases, the particular generated voltage (or current) would be independent of the connected circuit.

A ***passive element*** is one which does not generate electricity but either consumes it or stores it. *Resistors, Inductors* and *Capacitors* are simple passive elements. Diodes, transistors etc. are also passive elements.

Passive elements may either be *linear* or *non-linear.* Linear elements obey a straight line law. For example, a linear resistor has a linear *voltage vs current* relationship which passes through the origin (V = R.I). A linear inductor has a linear *flux vs current* relationship which passes through the origin (ϕ = k I) and a linear capacitor has a linear *charge vs voltage* relationship which passes through the origin (q = CV). [R, k and C are constants].

Resistors, inductors and capacitors may be linear or non-linear, while diodes and transistors are always non-linear.

Branch

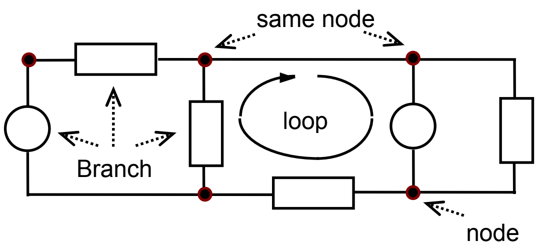
A *branch* represents a single element, such as a resistor or a battery. A branch is a 2 terminal (end) element.

Node

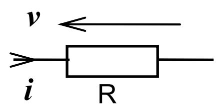
A *node* is the point connecting two or more branches. The node is usually indicated by a *dot* (●) in a circuit.

Loop or ***mesh***

A *loop* is any closed path in a circuit, formed by starting at a node, passing through a number of branches and ending up once more at the original node.



Resistance R [Unit: ohm (Ω)]



The relationship between voltage and current is given by $\boldsymbol{v = R\,i}$, or $\boldsymbol{i = G\,v}$, G = conductance = 1/R

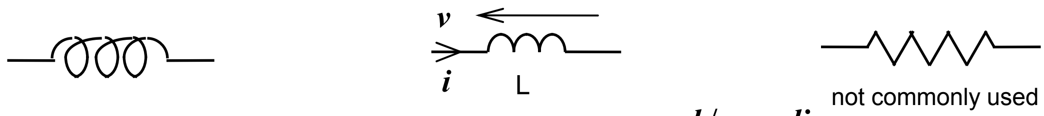
$$R = \frac{\rho\, l}{A}$$ where ρ is the resistivity, l the length and A the cross section of the material

Power loss in a resistor = $R\,i^2$. Energy dissipated in a resistor $w = \int R.i^2\,dt$

There is no storage of energy in a resistor.

Usage	conductor				semi-conductor			insulator			
Material	Silver	Copper	Gold	Aluminium	Carbon	Germanium	Silicon	Paper	Mica	Glass	Teflon
Resistivity (Ω m)	16.4×10^{-9}	17.2×10^{-9}	24.5×10^{-9}	28×10^{-9}	40×10^{-6}	0.47	640	10×10^{9}	0.5×10^{12}	10^{12}	3×10^{12}

Inductance L [Unit: henry (H)]



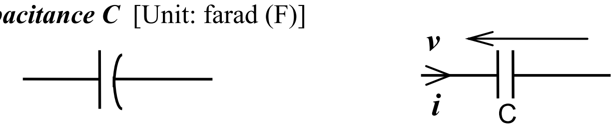
The relationship between voltage and current is given by $v = N\dfrac{d\phi}{dt} = L\dfrac{di}{dt}$

$L = \dfrac{N^2 \mu A}{l}$ for a coil; where μ is the permeability, N the number of turns, l the length and A cross section of core

Energy stored in an inductor = $\frac{1}{2} L\, i^2$

No energy is dissipated in a pure inductor. However as practical inductors have some wire resistance there would be some power loss. There would also be a small power loss in the magnetic core (if any).

Capacitance C [Unit: farad (F)]



The relationship between voltage and current is given by $i = \dfrac{dq}{dt} = C\dfrac{dv}{dt}$

$C = \dfrac{\varepsilon A}{d}$ for a parallel plate capacitor; where ε is the permittivity, d the spacing and A the cross section of dielectric

Energy stored in an capacitor = $\frac{1}{2} C\, v^2$

No energy is dissipated in a pure capacitor. However practical capacitors also have some power loss.

2.2 Fundamental Laws

The fundamental laws that govern electric circuits are Ohm's law and Kirchoff's laws.

Ohm's Law

Ohm's law states that the voltage v across a resistor is directly proportional to the current i flowing through it.

$v \propto i, \quad v = R\,.\,i$ where R is the proportionality constant.

A *short circuit* in a circuit element is when the resistance (and any other impedance) of the element approaches zero. [The term impedance is similar to resistance but is used in alternating current theory for other components]

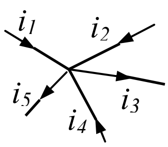
An *open circuit* in a circuit element is when the resistance (and any other impedance) of the element approaches infinity.

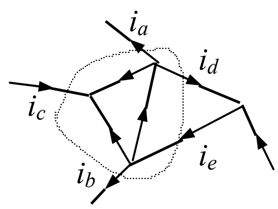
In addition to Ohm's law we need the *Kirchoff's voltage law* and the *Kirchoff's current law* to analyse circuits.

Kirchoff's Current Law

Kirchoff's first law is based on the principle of conservation of charge, which requires that the algebraic sum of the charges within a closed system cannot change. Since charge is the integral of current, we have *Kirchoff's Current Law* that states that the algebraic sum of the currents entering a node (or a closed boundary) is zero .

$\Sigma i = 0$





$i_1 + i_2 - i_3 + i_4 - i_5 = 0$

$-i_a - i_b + i_c - i_d + i_e = 0$

Kirchoff's Voltage Law

Kirchoff's second law is based on the principle of conservation of energy, which requires that the potential difference taken round a closed path must be zero.

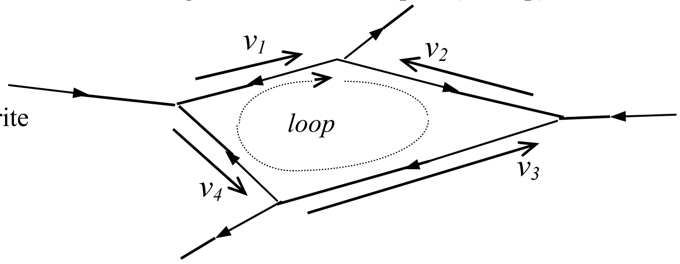
Kirchoff's Voltage Law states that the algebraic sum of all voltages around a closed path (or loop) is zero.

$$\Sigma v = 0$$

$$-v_1 + v_2 + v_3 + v_4 = 0$$

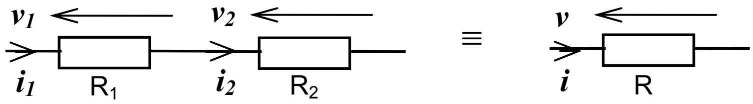
depending on the convention, you may also write

$$v_1 - v_2 - v_3 - v_4 = 0$$



Note: v_1, v_2 … may be voltages across either active elements or passive elements or both and may be obtained using Ohm's law.

Series Circuits



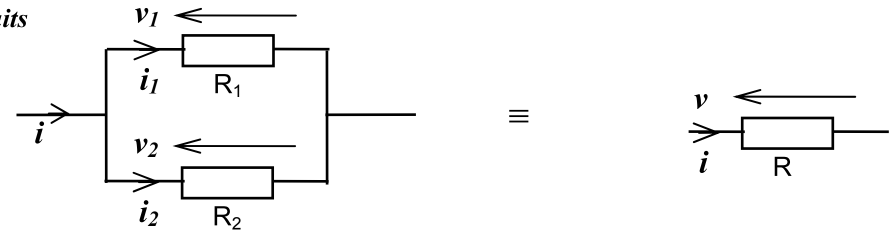
When elements are connected in series, from Kirchoff's current law, $i_1 = i_2 = i$ and from Kirchoff's Voltage Law, $v_1 + v_2 = v$. Also from Ohm's Law, $v_1 = R_1\, i_1,\ \ v_2 = R_2\, i_2,\ \ v = R\ i$

$$\therefore\ R_1\, i + R_2\, i = R\ i, \text{ or } R = R_1 + R_2$$

Also, $\dfrac{v_1}{v_2} = \dfrac{R_1\, i_1}{R_2\, i_2} = \dfrac{R_1\, i}{R_2\, i} = \dfrac{R_1}{R_2}$, and $\dfrac{v_1}{v} = \dfrac{R_1}{R_1 + R_2}$, $\dfrac{v_2}{v} = \dfrac{R_2}{R_1 + R_2}$ ……..voltage division rule

That is, in a series circuit, the total resistance is the sum of the individual resistances, and the voltage across the individual elements is directly proportional to the resistance of that element.

Parallel Circuits



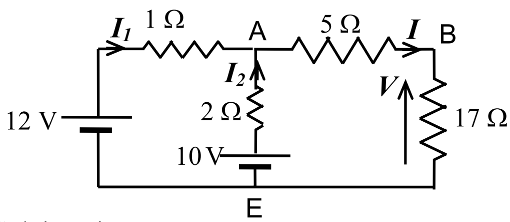
When elements are connected in parallel, from Kirchoff's current law, $i_1 + i_2 = i$ and from Kirchoff's Voltage Law, $v_1 = v_2 = v$. Also from Ohm's Law, $v_1 = R_1\, i_1,\ \ v_2 = R_2\, i_2,\ \ v = R\ i$

$$\therefore\ \frac{v}{R_1} + \frac{v}{R_2} = \frac{v}{R} \quad \text{or} \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad R = \frac{R_1\, R_2}{R_1 + R_2}$$

Also, $\dfrac{i_1}{i_2} = \dfrac{v_1/R_1}{v_2/R_2} = \dfrac{R_2\ v}{R_1\ v} = \dfrac{R_2}{R_1}$, and $\dfrac{i_1}{i} = \dfrac{R_2}{R_2 + R_1}$, $\dfrac{i_2}{i} = \dfrac{R_1}{R_2 + R_1}$ ……..current division rule

That is, in a series circuit, the total resistance is the sum of the individual resistances, and the voltage across the individual elements is directly proportional to the resistance of that element.

Example



Using Ohm's Law and Kirchoff's Laws

$$I = I_1 + I_2\,,$$

$$12 - 10 = 1\, I_1 - 2\, I_2,$$

$$10 = 2\, I_2 + 5\, I + 17\, I$$

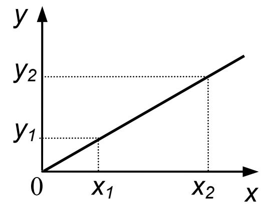
Solving gives

$I_1 = 1\text{A}$, $I_2 = -0.5\text{A}$, $\boldsymbol{I = 0.5\text{A}}$, $V_{AE} = 11\text{V}$, $\boldsymbol{V = 8.5\text{V}}$

2.3 Network Theorems

Complex circuits could be analysed using Ohm's Law and Kirchoff's laws directly, but the calculations would be tedious. To handle the complexity, some theorems have been developed to simplify the analysis. It must be emphasised that these theorems are applicable to *circuits* with *linear elements* only.

2.3.1 Superposition Theorem



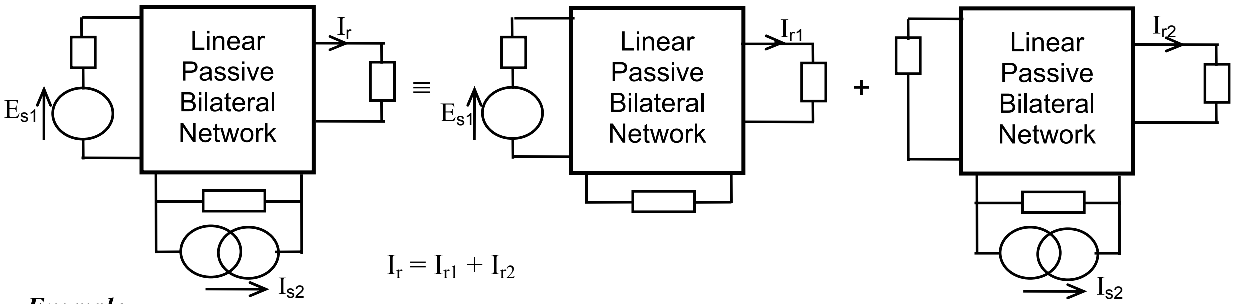
In a linear circuit, if independent variables x_1 gives y_1 and x_2 gives y_2

then, an independent variable $k_1 x_1 + k_2 x_2$ would give $k_1 y_1 + k_2 y_2$

where k_1 and k_2 are constants.

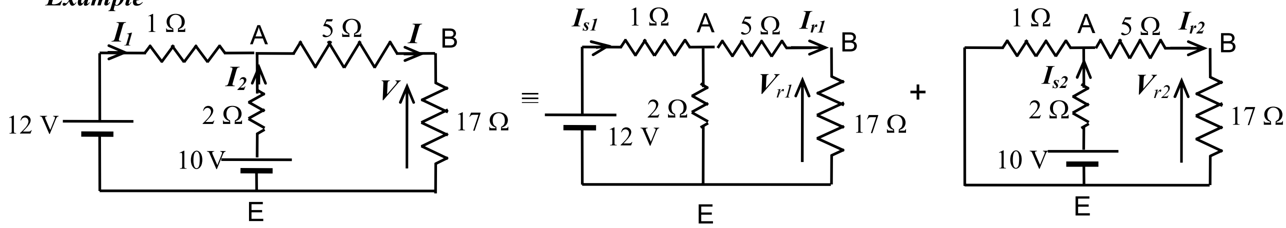
In the special case, when input is $x_1 + x_2$ the output would be $y_1 + y_2$

The *Superposition theorem* states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone [i.e. with all other sources replaced by their internal impedance.



$I_r = I_{r1} + I_{r2}$

Example



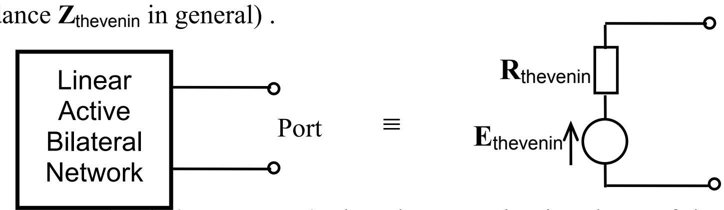
From sub-circuit 1, $I_{s1} = \dfrac{12}{1+2\,//\,(5+17)} = \mathbf{4.23529}\text{A}, \;\therefore\; I_{r1} = \dfrac{2}{2+(5+17)} \times \mathbf{4.23529} = \mathbf{0.35294}\ \text{A}$

similarly, $I_{s2} = \dfrac{10}{2+1\,//\,(5+17)} = \mathbf{3.38235}\text{A}, \;\therefore\; I_{r2} = \dfrac{1}{1+(5+17)} \times \mathbf{3.38235} = \mathbf{0.14706}\ \text{A}$

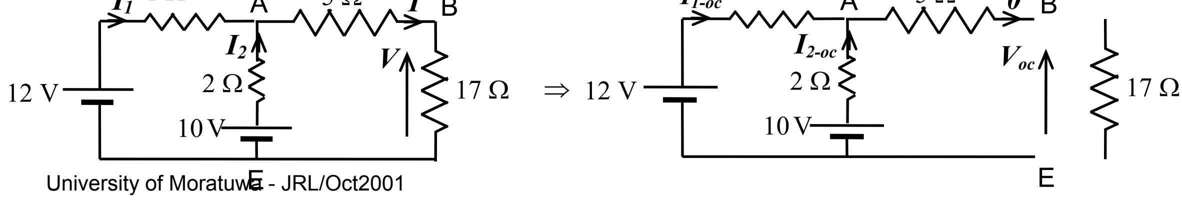
$\therefore$ from superposition theorem, $I = I_{r1} + I_{r2} = 0.35294 + 0.14706 = 0.50000 = \mathbf{0.5\ A}$ *(same result as before)*

2.3.2 Thevenin's Theorem

Any linear active bilateral single-port (a component may be connected across the 2 terminals of a port) network can be replaced by an equivalent circuit comprising of a single voltage source $\mathbf{E}_{\text{thevenin}}$ and a series resistance $\mathbf{R}_{\text{thevenin}}$ (or impedance $\mathbf{Z}_{\text{thevenin}}$ in general) .



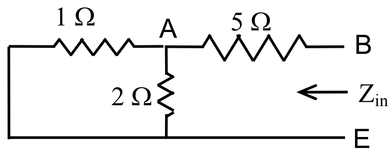
If the port is kept on open circuit (current zero), then the open circuit voltage of the network must be equal to the Thevenin's equivalent voltage source. If all the sources within the network are replaced by their internal resistances (or impedances), then the impedance seen into the port from outside will be equal to the Thevenin's resistance (or impedance).



If we are interested in determining V and/or I, open circuit the port BE as shown (temporarily disconnect 17 Ω resistor).

Then, $I_{1\text{-}oc} = -I_{2\text{-}oc} = (12 - 10)/(1 + 2) = 0.66667$ A , and $V_{AE\text{-}oc} = 12 - 1 \times 0.66667 = 11.33333$ V

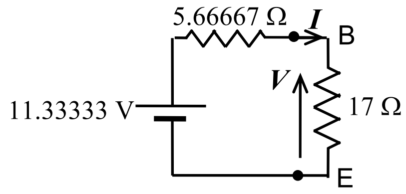
$\therefore$ $V_{\text{thevenin}} = V_{oc} = 11.33333 - 5 \times 0 = 11.33333$ V



With sources replaced by their internal resistances ,

$Z_{\text{thevenin}} = Z_{in} = 5 + 1//2 = 5.66667\ \Omega$

$\therefore$ original circuit may be replaced by the Thevenin's equivalent circuit shown.



$\therefore I = 11.33333/(5.66667+17) = 0.4999998 =$ **0.5 A** *(same as before)*

and $V = 17 \times 0.5 =$ **8.5 V** *(same as before)*

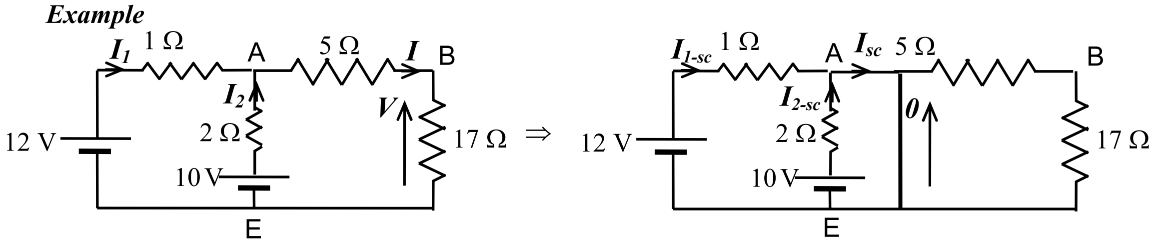
2.3.3 Norton's Theorem

Any linear active bilateral single-port network can be replaced by an equivalent circuit comprising of a single current source $\mathbf{I}_{\text{norton}}$ and a shunt conductance $\mathbf{G}_{\text{thevenin}}$ (or admittance $\mathbf{Y}_{\text{thevenin}}$ in general) .

Norton's theorem is the dual theorem of Thevenin's theorem where the voltage source is replaced by a current source .

If the port is kept on short circuit (voltage zero), then the short circuit current of the network must be equal to the Norton's equivalent current source. If all the sources within the network are replaced by their internal conductances (or admittances), then the admittance seen into the port from outside will be equal to the Norton's conductance (or admittance).

Example



Consider obtaining the equivalent circuit across the points AE. Short circuit AE as shown.

Then $I_{sc} = I_{1\text{-}sc} + I_{2\text{-}sc}$ and $I_{1\text{-sc}} = 12/1 = 12$ A, $I_{2\text{-sc}} = 10/2 = 5$ A, so that $I_{sc} = 12 + 5 = 17$ A

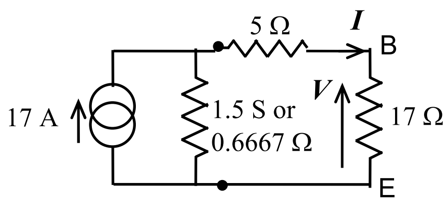
i.e. $I_{\text{norton}} = 17$ A.

Total conductance of a parallel circuit is the addition of the individual conductances.

$\therefore$ $G_{\text{norton}} = 1/2 + 1/1 = 1.5$ S

$\therefore$ Norton's equivalent circuit is

The current I is given by $\mathbf{17 \times \dfrac{0.6667}{0.6667 + 5 + 12} = 0.50002 = 0.5}$ A

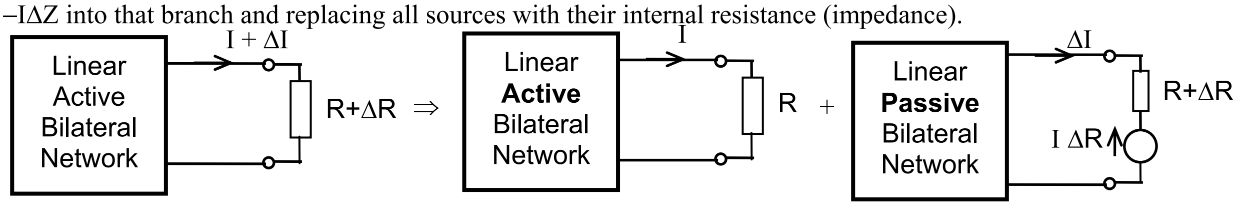


(same result as before)

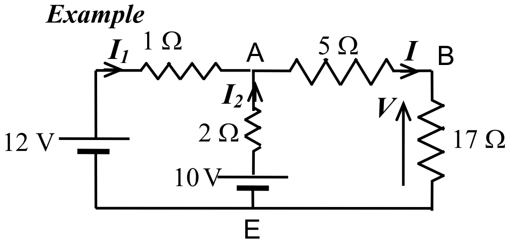
2.3.4 Compensation Theorem

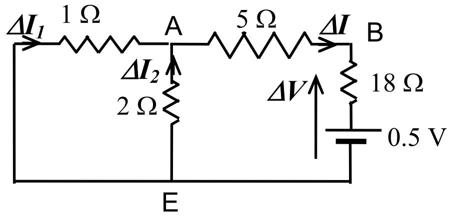
The compensation theorem is useful when one component in a circuit is changed by a small amount ΔZ to find the changes without recalculating the full network.

If the impedance of a branch in a network carrying a current I is changed by a finite amount ΔR (or ΔZ), then the change in the currents in all other branches of the network can be obtained by inserting a voltage source of –IΔZ into that branch and replacing all sources with their internal resistance (impedance).



Example





from earlier calculations, I = 0.5 A

Consider the change in I when 17 Ω resistor is changed by a small amount to 18 Ω.

ΔR = +1 Ω, $\quad$ $-I\,\Delta R$ = –0.5 × 1 = –0.5 V

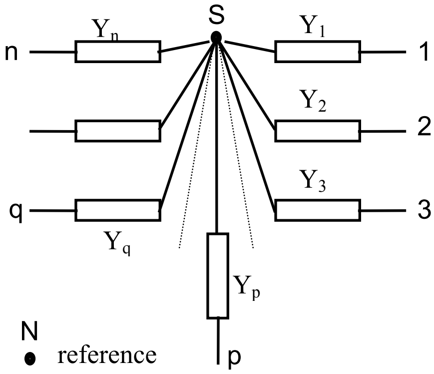
∴ changes in the current can be obtained from the circuit

$$\Delta I = -\frac{\mathbf{0.5}}{\mathbf{18+5+2 // 1}} = \frac{\mathbf{-0.5}}{\mathbf{23.6667}} = \mathbf{-0.02113}$$

∴ I = 0.5 - 0.02113 = **0.4789 A**

[As an exercise you may check this value using the other methods].

2.3.5 Millmann's Theorem



If a number of admittances Y_1, Y_2, Y_3 ….Y_p…. Y_q, …Y_n are connected together at a common point S, and the voltages of the free ends of the admittances with respect to a common reference N is known to be V_{1N}, V_{2N}, V_{3N} ….V_{pN}…. V_{qN}, …V_{nN}, then the voltage of the common point S with respect to the reference N is given as

$$V_{SN} = \frac{\sum_{p=1}^{n} Y_p V_{pN}}{\sum_{p=1}^{n} Y_p}$$

Proof is based on Kirchoff's Current Law at node S $\quad \sum_{p=1}^{n} I_p = 0$, and $I_p = Y_p\left(V_{pN} - V_{nN}\right)$

Example

Four resistances, 2 Ω, 1 Ω, 4 Ω, and 2.5 Ω are connected in star at a common point S across AS, BS, CS and DS. If the potentials of the other ends of the respective resistances with respect to earth E are V_{AE} = 100 V, V_{BE} = 80 V, V_{CE} = 60 V and V_{DE} =120 V, find the potential of the star point with respect to earth V_{SE}.

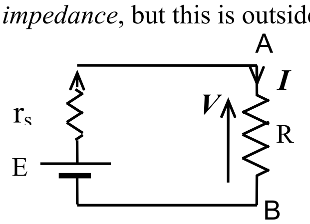
Using Millmann's theorem,

$$V_{SE} = \frac{\mathbf{1/2 \times 100 + 1/1 \times 80 + 1/4 \times 60 + 1/2.5 \times 120}}{\mathbf{1/2 + 1/1 + 1/4 + 1/2.5}} = \frac{\mathbf{50+80+15+48}}{\mathbf{0.5+1+0.25+0.4}} = \frac{\mathbf{193}}{\mathbf{2.15}} = \mathbf{89.77}\text{ V}$$

Note: Millmann's theorem can also be applied when a number of practical generators are connected in parallel to find the equivalent source. Thus theorem is also sometimes referred to as *the Parallel Generator theorem*.

2.3.6 Maximum Power Transfer Theorem

The maximum power transfer theorem states that the maximum power that can be supplied from a given source with internal resistance r_s to a purely resistive load R occurs when $R = r_s$. [A slightly different result occurs in the case of complex loads, but where the *Load impedance* becomes *equal* to the conjugate of the *source impedance*, but this is outside the scope of this course]



Proof: Power delivered to load = $P = V.\,I$, where $I = \dfrac{E}{r_s + R}$, $\quad V = R.I$

$$P = R.I^2 = \frac{E^2}{\left(R + r_s\right)^2}.R$$, for maximum power transfer to the load

$$\frac{dP}{dR} = 0 = \frac{E^2}{(R + r_s)^4} \cdot \left((R + r_s)^2 .1 - R.2(R + r_s)\right), \text{ or } R + r_s - 2R = 0, \text{ i.e. } R = r_s$$

Under this condition, it can be seen that $V=E/2$ and $P = E^2/4r_s$

How do we know that this corresponds to maximum power. We do not have to take the second derivative, but can reach that conclusion from physical considerations. We know that when $R = 0$ we have a short circuit (V=0) and no power is delivered to the source (of course a lot of power can come out of the source and get wasted), and when $R = \infty$ we have an open circuit (I=0) when again no power will be delivered. Therefore a physical maximum magnitude must occur in between these two values.

Example

A certain car battery has an open circuit voltage of 13.5 V and an internal resistance of 0.015 Ω. Determine the maximum power that the battery can supply to a load. Determine the voltage of the load under these conditions and the value of the resistance of the load.

Solution

From maximum power transfer theorem, the load resistance must equal the source resistance.

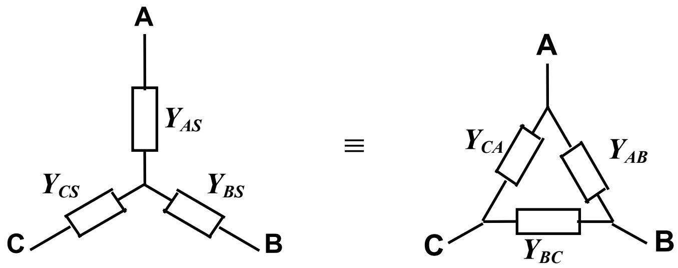
∴ load resistance = **0.015 Ω**

Maximum power that can be transferred = $13.5^2/(4\times0.015)$ = 2882 W = **2.882 kW**

Voltage across load under these conditions = 13.5/2 = **6.75 V**

In practice, it would not be acceptable for the voltage to drop to as low as 6.75 V for a car battery. We would expect that it would not drop much below 12 V. Thus the actual maximum power that can be obtained keeping other constraints would be significantly less than 2.882 kW.

2.3.7 Star-Delta Transformation



A *star connected* network of three admittances (or conductances) Y_{AS}, Y_{BS}, and Y_{CS} connected together at a common node S can be transformed into a *delta connected* network of three admittances Y_{AB}, Y_{BC}, and Y_{CA} using the following transformations.

$$Y_{AB} = \frac{Y_{AS}.Y_{BS}}{Y_{AS} + Y_{BS} + Y_{CS}}, \quad Y_{BC} = \frac{Y_{BS}.Y_{CS}}{Y_{AS} + Y_{BS} + Y_{CS}}, \quad Y_{CA} = \frac{Y_{CS}.Y_{AS}}{Y_{AS} + Y_{BS} + Y_{CS}}$$

Note: You can observe that in each of the above expressions if we need to find a particular delta admittance element value, we have to multiply the two values of admittance at the nodes on either side in the original star-network and divide by the sum of the three admittances.

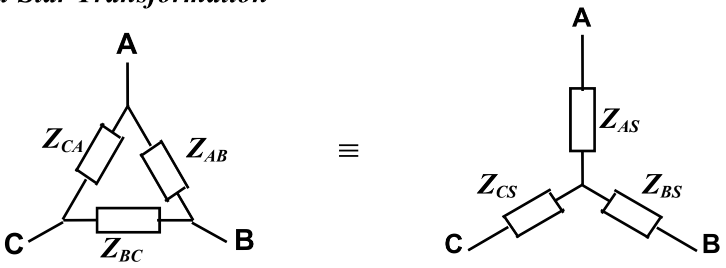
Nodal Mesh Transformation theorem

The nodal mesh transformation theorem is the more general transformation between a number of admittances connected together at a star point S and the corresponding mesh-connected network having many more elements. In this case, the equivalent mesh-admittance element Y_{pq} between two nodes p and q is as given. You can see that the star-delta transformation is a special case of the nodal mesh transformation. If there are n branches in the star-network, it can be easily seen that the mesh-network will have m=½ n(n-1) branches.

$$Y_{pq} = \frac{Y_{pS} \cdot Y_{qS}}{\sum_{r=1}^{n} Y_{rS}}$$

When n=3, m = ½ n(n-1)=3 but when n=5, m=10. It will be easily seen that the star and mesh networks will have the same number of elements only when n=3; otherwise m>n always. Thus the reverse process of transformation will only be possible when n=3. For this case only the Delta-Star transformation is also defined.

Delta-Star Transformation



A *delta connected* network of three impedances (or resistances) Z_{AB}, Z_{BC}, and Z_{CA} can be transformed into a *star connected* network of three impedances Z_{AS}, Z_{BS}, and Z_{CS} connected together at a common node S using the following transformations. [You will notice that I have used impedance here rather than admittance because then the form of the solution remains similar and easy to remember.]

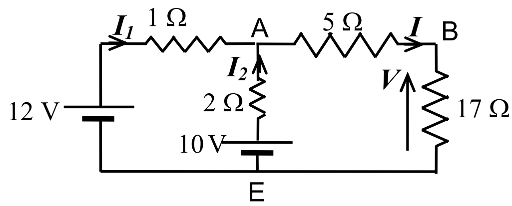
$$Z_{AS} = \frac{Z_{AB}.Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}, Z_{BS} = \frac{Z_{AB}.Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}}, Z_{CS} = \frac{Z_{CA}.Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

Note: You can observe that in each of the above expressions if we need to find a particular delta element value, we have to multiply the two impedance values on either side of node in the original star-network and divide by the sum of the three impedances.

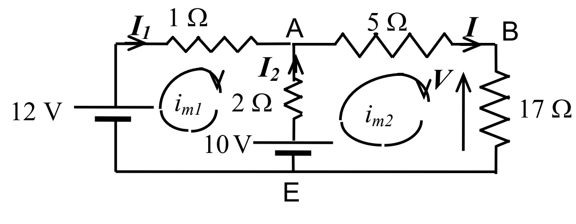
2.4 Introduction to Nodal and Mesh Analysis

When we want to analyse a given network, we try to pick the minimum number of variables and the corresponding number of equations to keep the calculations to a minimum. Thus we would normally work with either currents only or voltages only. Let us consider an example to illustrate this.

2.4.1 Mesh Analysis



The usual practice for a network such as this is to mark only two independent currents I_1 and I_2 and the other current I would become a dependent variable (based on Kirchoff's current law). Then we write down the Kirchoff's voltage law equation for the two identified loops. [This is how we solved it in the first place]. Mesh analysis makes use of this, but marks currents in a different manner.



In mesh analysis, we mark independent mesh currents i_{m1} and i_{m2} as shown. The branch currents can all be expressed in terms of this mesh currents.

$$I_1 = i_{m1}, I_2 = -i_{m1} + i_{m2}, I = i_{m2}$$

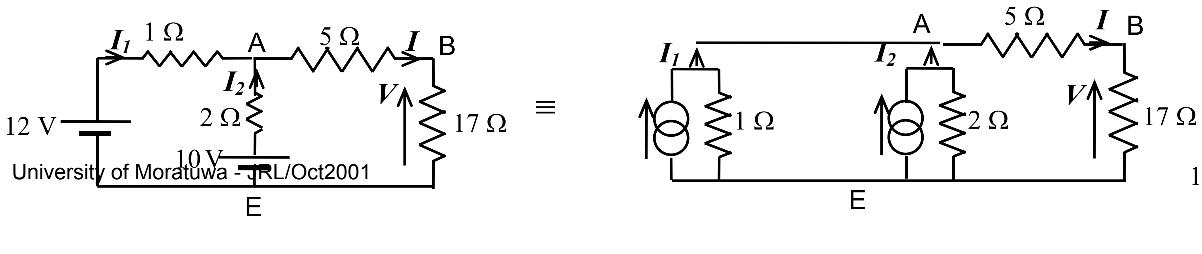
Kirchoff's voltage law equations are written in terms of these mesh currents

$$12 = 1\, i_{m1} + 2(i_{m1} - i_{m2}) - 10 , \quad 10 = -2(i_{m1} - i_{m2}) + 5\, i_{m2} + 17\, i_{m2}$$

These equations are solved in the usual manner to give the mesh currents. Using the mesh current the branch currents may be determined.

2.4.2 Nodal Analysis

In nodal analysis, basically we work with a set of node voltages. The voltage sources would also usually be replaced by equivalent current sources. Consider again the earlier circuit. This can be replaced by the circuit shown alongside. The node E would usually be taken as the reference and given a potential of zero.



$\frac{12}{1}$ A $\frac{10}{2}$ A

Let V_{AE} and V_{BE} be the potentials (or voltages) of A and B with respect to the reference E. From these voltages and using Ohm's law the currents in the individual resistors (1 Ω, 2 Ω, 5 Ω and 17 Ω) can be written as $V_{AE}/1$, $V_{AE}/2$, $(V_{AE} - V_{BE})/5$ and $V_{BE}/17$.

Applying Kirchoff's current law to node A and to node B, we have

$12/1 - V_{AE}/1 + 10/2 - V_{AE}/2 - (V_{AE} - V_{BE})/5 = 0,$ also $(V_{AE} - V_{BE})/5 = V_{BE}/17$

i.e. $17 - V_{AE} - V_{AE}/2 - V_{AE}/5 + V_{BE}/5 = 0,$ also $V_{AE}/5 - V_{BE}/5 = V_{BE}/17$

These equations may be solved to give the node voltages at A and B. The branch currents can then be obtained.

In fact we need not even take node B, but take the branch (5+17) as connected to A.

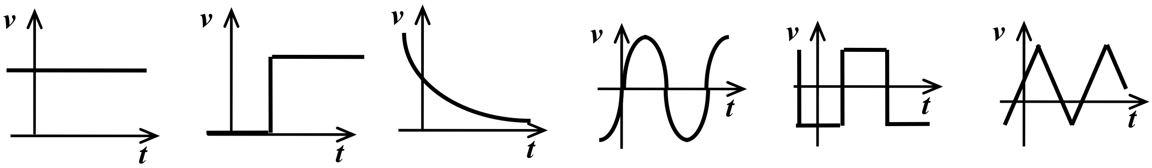
$12/1 - V_{AE}/1 + 10/2 - V_{AE}/2 - V_{AE}/22 = 0,$ or $\mathbf{17 - V_{AE}(1 + 0.5 + 0.04545) = 0,}$ i.e. $\mathbf{V_{AE} = 11.0000\ V}$

This is the same result we had in the first example with Ohm's law and Kirchoff's laws and you can see that only one equation was required to obtain the answer.

Both the Mesh Analysis and Nodal Analysis theory is usually built up using matrices so that they may be used for analysis on the computer. However this section is considered to be beyond the scope of this course.

2.5 Introduction to Waveform Analysis

Waveforms of voltage and current can take various forms. They may take a constant dc value (figure a), a step waveform (figure b), an exponentially decaying shape (figure c), a sinusoidal waveform (figure d), a rectangular waveform (figure e), a triangular waveform (figure f) and many other shapes.



a b c d e f

You will notice that waveforms **a, b** and **c** are unidirectional, where as **d**, **e** and **f** have positive and negative values. You will also notice that **d**, **e** and **f** are repetitive waveforms (periodic). Also **d** and **e** have mean values which are zero, where as **f** has a positive mean value. Repetitive waveforms can always be represented by a combination of waveforms with mean value zero (alternating component) and with a positive or negative mean value (direct component).

The **peak value** of a waveform is not indicative of its useful value. On the other hand a non-zero waveform can have a zero **mean value**.

$$I_{mean} = \frac{1}{T}\int_0^T i(t).dt$$

and $$I_{average} = \frac{1}{T}\int_{positive} i(t).dt - \frac{1}{T}\int_{negative} i(t).dt$$

Thus the mean value alone is not useful. One method commonly used is to invert any negative part of the waveform and obtain the **average value** of the **rectified** waveform. This too is not fully indicative of the useful value. The useful value or **effective value** of a alternating waveform is the value with which the correct value of power can be obtained.

$$R.I_{eff}^{\ 2}.T = \int_0^T R.i^2(t).dt$$

or $$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2(t).dt}$$

We can see that the effective value is obtained by taking the square **root** of the **mean** of the **square**d waveform. Because of the method of obtaining this value, it is usually called the **root-mean-square** value or **rms** value.